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Random Scattering of Electrons in Magnetic Fields*

1. An Experiment

Untypically for a theorist, let me begin my talk with an experiment. Consider a tiny golden ring with a diameter of only 820 nm and a thickness of 40 nm. A greatly magnified picture of such a ring is given in Fig. 1. The ring is coupled with four contacts to the surrounding bulk material. These contacts or "leads" serve the purpose of injecting current into the probe and measuring the resulting voltage difference. It goes without saying that highly elaborate fabrication techniques are necessary to manufacture such a microstructure. The fabrication process usually involves the use of a computer-steered electron microscope and samples of the quality shown in Fig. 1 have become available only comparatively recently, i. e. since the beginning of the eighties. In 1985, a group of physicists at the IBM Thomas J. Watson research center in New York performed an experiment where they put this very ring into a static, homogeneous magnetic field, cooled it down to 0.04 K (four hundredth of a degree above the abolute zero of temperature), and measured its resistance as a function of the magnetic field strength. The resulting curve is shown in Fig. 2. Two prominent features of this curve catch the eye. While the overall behaviour is dominated by irregular fluctuations, local magnifications at arbitrary field strengths exhibit a rather regular, oscillatory shape. Interestingly, this curve is reproducible: Sweeping several times over the full range of magnetic field strengths the resistance will always follow the same highly complex pattern. Being a unique property of the specific sample such patterns have been called "magnetofingerprints".

In view of the complexity of the experimental curve it is certainly senseless to investigate every detail of the graph in Fig. 2. Instead, one should look for the generic properties of the data and, to put it in Pier Mello's words, "ask the right questions".

The "right questions" are concerned with aspects of the experiment that are independent of the particular sample under investigation such as the

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origin, the amplitude, and the magnetic field scale of the two main features of the curve. In the following, I will focus my attention on these issues.

2. Further Motivation

I claim that the IBM experiment demonstrates the *quantum mechanical nature* of the electron. One may, of course, legitimately ask why this is interesting and why one should bother to deal with these questions. The answer, I believe, is twofold. First, there is considerable *scientific* interest associated with the problem. Quantum mechanics is a theory originally developed for atoms and objects of similar size. The IBM experiment shows that this microscopic theory becomes relevant at unexpectedly large length scales (we have to keep in mind that 11.tm corresponds to a few thousand atomic distances). At these length scales the experimental physicist has comparatively direct access to several interesting quantities like, e. g., the resistance. Therefore experiments like the one performed at the IBM laboratory provide a sensitive test of our understanding of both quantum coherence (we will come back to this term later) and transport properties. This is my main personal motivation to investigate systems like the one in Fig. 1.

Second, research in this field may become very important for *technological* reasons. We are all aware of the tremendous progress in computer and chip design which has revolutionized our daily life over the past few decades. Highly integrated circuits have succeeded in reducing the costs and increasing the speed of the machines that were built from them. Unfortunately, within ten years or so, present-day computer technology based on the Silicium-MOSFET *(metal oxide semiconductor field effect transistor)* will have practically reached its limit. The reason is simple: a MOSFET has a minimal size. Upon further miniaturization the MOSFET loses the properties that were the basis of its proper functioning. Do we have promising new strategies to improve computer performance? Much effort has been invested in trying to replace Silicium by Galliumarsenide (GaAs), a material which allows for considerably higher speed. However, GaAs is expensive and difficult to handle and the prospects are at best unclear. A completely different strategy places emphasis on the development of machines with many parallel processors. This is certainly a very fruitful approach but it does not lead to further miniaturization. Currently, so-called exotic quantum effects are being discussed as the basis for novel ultrasmall quantum devices which might revolutionize microelectonics in a similar manner as the transistor revolutionized ordinary electronics. Optimistic scientists do not hesitate to dream of a supercomputer on a single chip. To pursue this kind of program, much research concerning the electronic behaviour of very small devices is called for. The IBM experiment described above exhibits at least one of those exotic quantum effects.

Furthermore, the problem I want to address in this talk is far more general than it might seem now. Also, there are certain connections to what is usually abbreviated as "quantum chaos". I will return to both of these points at the end of my presentation.

3. Constructing a Model

The first thing we have to take into account when we try to understand electron transport in solids is the regular crystal lattice of atoms. It was Felix Bloch in 1928 who formulated what is since known as the Bloch theorem. This theorem essentially states that electrons in a perfectly periodic solid propagate similarly to free electrons with certain modifications due to the lattice. Therefore we cannot expect such spectacular effects as in Fig. 2 from the interaction between electrons and periodic lattice. However, in reality there is no such thing as a perfect crystal. There are always defects in the crystal order or atoms of a different element irregularly distributed over the probe. I will refer to these deviations from perfect order collectively as "impurities". The distribution of these impurities depends on the history and the manufacturing process of the sample. For all practical reasons it can be considered to be random. The point of central importance is that the impurities act as obstacles for the electrons in the probe. In 1958 Philip Anderson emphasized the prominent role of these obstacles by focussing attention on the impurities alone, neglecting the perfectly ordered crystalline lattice. We will adopt this view in the following considerations.

Let us now formulate the theoretical model for the real physical system in Fig. 1. We consider a ring of finite thickness with one external lead on either side to inject and/or extract the current. Inside the body of the ring we assume a certain distribution of scattering centers (impurities). The whole ring is placed in a perpendicular, homogeneous magnetic field and the only interaction taken into account apart from the influence of the magnetic field on the electrons is the one between electrons and impurities. In view of this rather primitive model one may be tempted to ask whether we are dealing with a simple or even trivial problem. The answer to this question again has two parts. First, let us restrict ourselves to a purely technical level. We cannot calculate the conductance or, equivalently, resistance of our model for a particular fixed distribution of the impurities due to the vast number of coordinates introduced by specifying all the individual impurity positions. Instead, one is led to consider an *ensemble* of impurity distributions (this is the statistical input into the model) and calculate statistical measures like the mean conductance or the variance of the conductance. The necessary averaging process is technically difficult and several mathematical methods have been developed for this purpose. Today, this problem can be considered to have been solved to a certain extent. Second, our model of course oversimplifies the real situation. We neglected the presence of additional interactions such as those between electrons and the lattice vibrations (phonons) or between the electrons themselves. The main reason why the IBM experiment was performed at very low temperatures was to avoid electron-phonon scattering. We will come back to this point later. To what extent the electron-electron interaction is important for understanding the properties of small metallic systems is presently being actively discussed.

4. Treating **the Model**

In classical physics the motion of, say, a point particle is given by Newton's equation. In quantum mechanics the notion of a point particle with well-defined position *and* momentum is abandoned and replaced by the so-called wave function governed by the Schrödinger equation. A wave function associates with every point in space a complex number *c*. Let us represent this complex number (which can be viewed as a *pair* of real numbers) by an arrow from the origin to some point in the plane. The x-coordinate of the arrow corresponds to one of the two real numbers (the "real" part, say) and the y-coordinate to the other (the "imaginary" part). Alternatively, we may characterize the complex number by the length of the arrow (the modulus $|c|$ of *c)* and the angle between the arrow and the positive x-axis (the phase cp). All wave phenomena, not only the wave function of quantum mechanics, can be represented in such a way. Superposition of two waves amounts to combining, for every point in space, the two arrows according to the rules of vector addition. The resulting field of arrows represents a new wave. The important role of the wave function is due to its physical interpretation: The squared length $|c|^2$ of the arrow at any given point is the probability of observing the particle (in our case an electron) at this same point.

We cannot rigorously solve the Schrödinger equation here. But we can discuss an intuitive picture emerging from the rigorous solutions. Let us for reasons of simplicity consider a rectangular sample with a certain fixed impurity distribution. We disregard the magnetic field for the moment. We can visualize the electrons as plane waves propagating through the leads connected to our probe and eventually impinging on the impurities. As a consequence, a circular wave is formed around each single impurity and the resulting interference pattern soon becomes extremely complicated. An alternative and equivalent visualization is more suitable for our purposes here. Instead of wave fronts we now employ *paths* to characterize the electron's motion. Unfortunately, this picture is no less complicated than the previous one since we have to take into account *all possible* paths of the electron. When an electron path hits an impurity we have to allow for a continuation of this path in all possible directions. This corresponds to the circular wave of the previous picture. Whenever two paths meet, their superposition is calculated as described above by adding the two complex numbers at this point. The probability of an electron moving from A to B is given by the superposition of all possible paths connecting these two points. Two oberservations are of central importance for us. First, the probability of an electron penetrating the whole sample — a quantity closely connected to the conductance — is given by the superposition of very many complicated paths. Second, the values of the phase φ along the path depend only on the geometry of the path. To explain this last statement a little further we come to the notion of *quantum coherence.*

Let us fix two particular paths which originate from the same point at the left border of the rectangular sample and end at a common point on the right side. In between, however, they differ from each other. We assume in a first step that the second of our statements above is true, i. e. the phases depend only on the geometry of the path. Then, whenever these two paths contribute to the description of some electron's motion, the relative phase difference $\Delta\varphi = \varphi_1 - \varphi_2$ between them is the same. Therefore the superposition of the complex numbers c_1 and c_2 associated with the two paths leads to the transmission probability

$$|c_1 + c_2|^2 = |c_1|^2 + |c_2|^2 + 2|c_1||c_2| \cos(\Delta\varphi).$$

This is called a coherent superposition. We note the dependence on the phase difference $\Delta\varphi$. If, on the contrary, we allow for time-dependent effects like lattice vibrations, the relative phase $\Delta\varphi$ becomes arbitrary. Depending on whether or not the electron interacts with a phonon (a process which changes the phases along the relevant paths), the final phase difference will assume any possible value. Since $\cos(\Delta\varphi)$ can either be positive or negative this means that the corresponding term does not effectively contribute at all and we have

$$|c_1 + c_2|^2 = |c_1|^2 + |c_2|^2.$$

This defines an *incoherent* superposition. A sample for which the experimental conditions are chosen such that only coherent and no incoherent superpositions can occur is called a *mesoscopic system.* The **IBM** experiment had to be performed at very low temperatures to reduce the lattice vibrations and thereby ensure that the conditions for the mesoscopic regime were actually met.

Having dealt with impurity scattering we turn to the effect of a magnetic field. The simplest case is that of a metallic loop having two contacts to the right and to the left with a magnetic flux through the opening. We disregard impurities and the magnetic field does not penetrate the body of the loop. Electrons injected into this loop have the choice of propagating through the upper or lower half, i. e. there are exactly two possible paths. Due to the famous Aharonov-Bohm effect (discovered in 1959), the corresponding relative phase Aço acquires an extra contribution determined by the magnetic flux. Under ideal conditions the transmission through the loop is turned on and off as one varies the magnetic field strength forcing the two electron paths to interfere either constructively or destructively.

Before coming back to the original **IBM** experiment, we consider in an intermediate step the effect of a magnetic field on the transport properties of a rectangular sample. We have already said that the conductance is determined by a superpositon of very many paths. Since we deal with a mesoscopic system, this superposition sensitively depends on the phases of all these paths. If we change the magnetic flux through our sample, the phases will readjust accordingly, the details of the interference will be altered, and the final result (the conductance) will differ slightly from its previous value. Due to the high number and complexity of the electron paths involved, these changes will not follow a regular pattern but appear to be rather irregular and erratic. These are the famous *universal conductance fluctuations* discovered in 1984.

After all these considerations understanding the IBM experiment is a simple matter. Instead of a rectangle we are dealing with a ring of finite thickness in a homogeneous magnetic field. A certain amount of flux penetrates the body of the ring and induces universal conductance fluctuations in the way just described. The flux through the opening of the ring gives rise to the Aharonov-Bohm effect and the ensuing oscillatory behaviour of the conductance or, as in the case of the IBM experiment, resistance. In principle, this quantum effect could serve to construct very small switches. The field scales, the typical variations of the magnetic field strength over which the oscillations or the irregular fluctuations occur, depend on the ratio of the magnetic flux in the ring and in the opening, i. e. on the geometry of the ring.

5. Conclusion and Outlook

I have promised to comment on the generality of the mechanisms discussed in my talk. In principle this talk could have been entitled "Wave Transport through Disordered Media". The transport of electrons through disordered solids is only one example. Many of our considerations also apply to such diverse processes as the penetration of light through regions with randomly varying index of refraction, of radio waves through fog, and of sound waves through randomly distributed geological structures in the earth. The last item plays a certain role in the search for gas and oil reservoirs. Of course, all these problems have their own peculiarities and difficulties. But the underlying physics, the scattering of waves from random obstacles, guarantees a great degree of similarity between them.

Finally, I would briefly like to point out three recent developments connected with the topic of my talk. First, let us recall the views of Bloch (perfectly regular crystal) and Anderson (totally disordered solid), respectively. Both views are rather extreme and it is very natural to expect that a combination of them gives rise to yet another variety of unexplored phenomena. Research in this direction has been intensifying in the last few years. Second, instead of investigating transport properties of *open systems* one can think of disconnecting a mesoscopic system from the external world and ask for the consequences of quantum coherence in *isolated* systems. The problem of the so-called *persistent currents* belongs to this category. Considerable progress concerning these questions has been made by the work of the physics group at the Wissenschaftskolleg. Third, much effort is currently spent manufacturing samples with few or no impurities. Such devices are called *ballistic.* They are realizations of quantum billiards and can serve to investigate the quantum analogue of classical chaos.

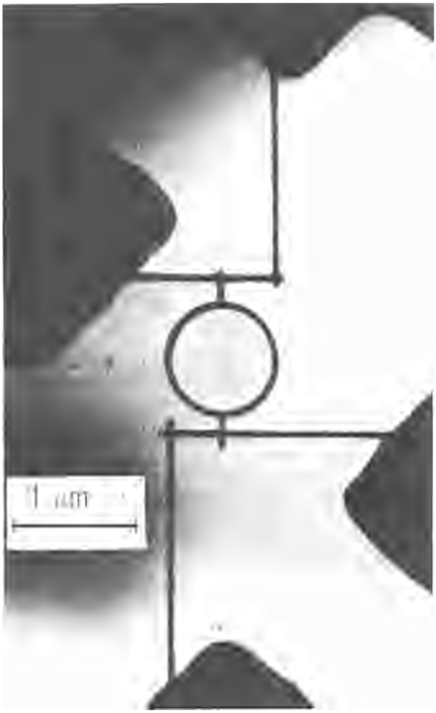

Fig. 1

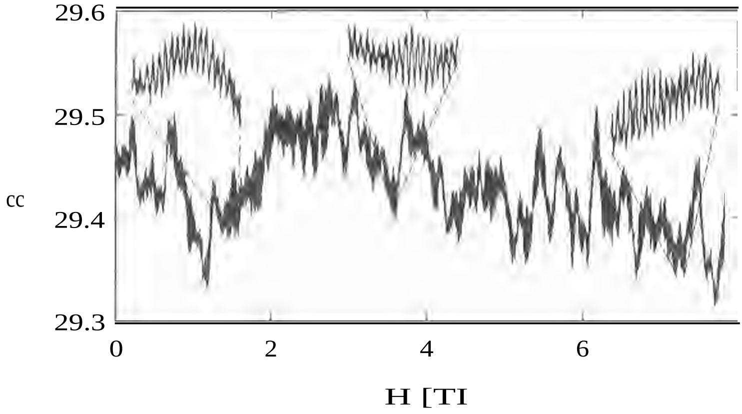

Fig. 2