Oriol Bohigas

Chaotic Dynamics, Travelling Salesmen and Lectures for Non-Physicists

Born in Barcelona in 1937. Undergraduate studies in physics until 1962 at the University of Barcelona, followed by graduate studies at Orsay (Paris). Since 1964 he has worked at the Institut de Physique Nucléaire, Orsay, France, as a theoretical physicist, Directeur de Recherches at the Centre National de la Recherche Scientifique (CNRS). Visiting Professor at the University of Rochester (USA), Lawrence Berkeley Laboratory, Universidad Nacional Autonoma de Mexico, Physical Research Laboratory (Ahmedabad, India), Yukawa Institute (Kyoto, Japan), Universidad Autonoma de Madrid (Spain), MPI für Kernforschung Heidelberg (Germany); Alexander von Humboldt Prize (1992). Fields of research: Structure of atomic nuclei, nuclear spectroscopy, collective motions, giant resonances, statistical properties of nuclei. Present interests: Relationship between classical chaotic dynamics and quantum mechanics, applications of random matrix theories. — Address: Division de Physique Théorique, Institut de Physique Nucléaire, 91406 Orsay Cedex, France.

During my eight-month stay at the Wissenschaftskolleg (December 1992 - July 1993), I was involved in three different spheres of activity: 1) continuation of research already started prior to my arrival at the Kolleg, 2) activity related to the presence at the Kolleg of a small coherent group of theoretical physicists (P. Mello, A. Müller-Groeling, T. Seligman, U. Smilansky, H. Weidenmüller and M. Zirnbauer) sharing common interests, namely quantum chaos and the physics of mesoscopic systems, 3) activity motivated to a large extent by the general structure and atmosphere of the Kolleg.

1) Continuation of current research

I have been working on two topics:

a) Chaotic dynamics and quantum tunnelling
(in collaboration with D. Boosé, R. Egydio and V. Marvulle)[1]

It is important to identify physical phenomena in the quantum regime for which the regular or chaotic nature of the underlying classical dynamics is relevant. Tunnelling, a classically forbidden phenomenon, may provide such a case. For this purpose, we have studied the motion of a free particle in a box which bounds elastically from the walls (billiards). The box is a two dimensional flat surface bounded by two non-concentric circles (a non-concentric annular box). This system depends on two real parameters: the radius of the interior disk and the eccentricity. By changing the parameters, several interesting limiting regimes can be obtained: integrable, fully chaotic, mixed dynamics. What makes this system particularly appealing is that it is possible to change the parameters of the model but keep a part of the regular phase space undisturbed. This, in turn, is reflected in the quantum regime by the presence of almost undisturbed quasidoublets in the spectrum. And the exponentially small splittings of these quasidoublets reflect the more or less chaotic nature of the phase space separating the regular tori. We have established the importance of the role of chaotic dynamics in the properties of the splittings. Despite extensive numerical calculations, we can presently give only a qualitative description. A theory of the tunnelling in the presence of chaos remains to be developed. I should mention that these studies have motivated experiments with microwave cavities by A. Richter and collaborators at Darmstadt and CERN. A. Richter (Darmstadt) visited the Kolleg to discuss, among other issues, these experiments with us.

b) Chaotic dynamics and the statistical properties of roots of random polynomials
(in collaboration with E. Bogomolny and P. Leboeuf)[2]

In the context of semiclassical approximations for multidimensional quantum systems and the manifestations of chaotic behaviour in quantum mechanics, one quite often needs to find the roots of polynomials of high degree whose coefficients are rapidly-varying erratic functions of the

[1] 0. Bohigas, D. Boosé, R. Egydio, V. Marvulle, »Quantum tunnelling and chaotic dynamics«, *Nucl. Phys. A* 560 (1993) 197

[2] E. Bogomolny, 0. Bohigas, P. Leboeuf, »Chaotic Dynamics and the Statistical Properties of Random Polynomials«, preprint, submitted for publication

energy. This means that even in a small interval of energy they can be considered as random variables. Though the distribution of the roots of random polynomials have been intensively studied in the past, its relevance with respect to the specific problems which naturally arise in the context of quantum chaotic dynamics (and in other domains of physics as well) have been underestimated. We explore this connection.

It should be remembered that the use of a statistical approach in the description of complex systems is an old idea. In particular, the random matrix theory, originally formulated in the context of nuclear physics, has had great success and impact in the context of quantum chaotic dynamics and disordered systems. Random polynomials may provide a complementary statistical approach to random matrix theories.

More specifically we investigate the distribution of the roots of polynomials of high degree with random coefficients. It is shown that under quite general conditions their roots tend to concentrate near the unit circle in the complex plane. In order to further increase the unitarity, we study in detail the particular case of self-inversive random polynomials and show that for them a finite portion of all roots lies exactly on the unit circle. Correlation functions of these roots are also computed analytically, and compared to random matrix theories. The problem of the ergodicity of chaotic wave-functions is also considered. For that purpose we introduce a family of random polynomials whose roots spread uniformly over the phase space. While these results are consistent with random matrix theory, they provide a new and different insight into the problem of quantum ergodicity. Special attention is devoted to the role of symmetries in the distribution of roots of random polynomials.

A two week invitation to the Kolleg of P. Leboeuf (Orsay) allowed us to make the final adjustments to the manuscript.

2) Activity related to the presence of the physics group at the Kolleg

Besides my participation, as a speaker or as a listener, in our weekly discussion group seminar at the Kolleg, two projects got off the ground during my stay in Berlin.

a) New evidence of GOE statistics for compound nuclear resonances
(in collaboration with M. Lombardi and T. Seligman)[3]

The nuclear data ensemble was compiled in the past by Haq, Pandey and Bohigas. The statistical fluctuations of level sequences, mainly coming

from compound nuclear resonances, was found to be in excellent agreement with the GOE (Gaussian Orthogonal Ensemble) prediction. In the present work we analyze the nuclear data ensemble according to new, extremely sensitive tests. One concerns level statistics and is based on a theorem by Dyson and Mehta which relates properties of a GOE sequence with properties of eigenvalues of the sympletic ensemble. The other concerns the absence of correlations between energy levels and intensities. When these new very sensitive tests are used, it is found again that random matrix predictions and data are consistent. Though expected, this is a significant result, which improves upon previous ones.

b) Some new aspects of the travelling salesman problem
(in collaboration with B. Berg)

This is a famous problem where again regularity may be expected to emerge out of chaos. The travelling salesman has to visit all his customers, each in a different city, in such a way that the distance (cost) is minimized. The cities are randomly scattered over a finite domain in the plane. The number of possible routings increases exponentially with the number of cities visited, rendering impossible comparison of all the possibilities. However, there are algorithms inspired by statistical mechanics which allow one to efficiently compute itineraries which are close to the optimum solution. We suggest that new statistical approaches may provide insight to characterize the close-to-the-optimum solutions. Specifically, we have considered the probability distribution of the distances between successive cities visited, and the probability distribution that the distances between successive cities visited is the first, second, third ... nearest neighbour city. Both distributions seem to rapidly approach asymptotic distributions in the limit of a large number of customers. We also consider the role of the topology of the domain in the characterization of the solution as well as the increase beyond two of the dimensions of the space.

A two week invitation of R. **Jalabert** (Orsay) to the Kolleg was useful to discuss several problems of mesoscopic systems, in particular the orbital magnetism of small metallic particles in the ballistic regime, motivated by recent measurements by L. Lévy at AT & T. It turns out that using semiclassical methods it is possible to work out expressions for the Landau diamagnetism and the de Haas-van Alphen effect for finite domains.

During my stay at the Kolleg I prepared two guest lectures, one to be held at the Symposium "From Spectroscopy to Chaos" honoring J. B.

[3] M. Lombardi, O. Bohigas, T. Seligman, »New evidence of GOE statistics for compound nucleus resonances«, submitted to *Phys. Lett. B*

French, delivered in Rochester in April 1993, and another at the Conference "Perspectives in Nuclear Physics", held in Copenhagen in June 1993. I also hosted a seminar at the Hahn-Meitner Institute in Berlin.

3) Some "genuine" fellow activity

A long-term stay at the Kolleg may offer a rather unique opportunity to open one's own mind, to break down intellectual prejudices, and begin to explore new territories. In this respect, I will briefly comment on some of my individual experiences.

There exists in our day, a school of thought, mainly among theoretical physicists, which aims to bridge, as much as possible, the path between physico-chemical sciences and life sciences. How far have we come since, for instance, the book by E. Schrödinger " *What is life?"* published in 1944? Is the science of complexity and non-linear dynamics usefully permeating some aspects of the life sciences? Should we, senior physicists, encourage young generations to go in the direction of fields like modelling and analysis of networks, brain activity, pattern formation and wave propagation in biological systems, the theory of evolution? In this respect, the presence at the Kolleg of several distinguished biologists was most informative to me, through seminars, private discussions and recommended readings. By now, I probably have a more general, as well as more critical view of this whole fundamental area which appears to me in many respects a fascinating but, also, in many others, as deceptive.

Reciprocally, a stay at the Kolleg also offers the rather unique opportunity of communicating with scholars working in fields other than one's own, enabling one to see beyond one's own motivation, findings, and the significance of one's own work. The rules for that are rather unusual, because you are not allowed to use technical language nor professional jargon. How far one can go in this direction of reflection and vulgarisation is certainly challenging. But one should bear in mind that pedantry, frivolity and dilettantism are the obvious traps to be avoided.

In this direction, I gave two seminars during my stay in Berlin. One at the Einstein-Forum in Potsdam, where a day of interdisciplinary discussions on Chaos was organized, the other at the Kolleg, where I borrowed from Wigner the title of my talk: "The unreasonable effectiveness of mathematics in the natural sciences". The preparation of these seminars was most challenging to me. As a result, I am presently preparing a set of five or six hour lectures in which I pretend to communicate to non-physicists what is the object of theoretical physics, the interplay between physics and mathematics, the to a large extent non-linear evolution of physical

ideas from a historical perspective. Maybe one day in the near future I will have an occasion to deliver such lectures. I would like to mention the most effective assistance provided by the librarians of the Kolleg in preparing these lectures.

Last but not least, I want to mention how stimulating life in Berlin is. Particularly, as far as I am concerned, the musical life. Do we, scholars at the Wissenschaftskolleg, in our interrogations ever even aspire to the level attained at the Komische Oper by Harry Kupfer *(„Ich möchte alle Fragen der Welt in dieser schönen totalen Kunstform, der Oper, durchspielen") ?*