Gérard Maugin

Solitons, Pseudomomentum, Fracture, Thermodynamics and All That

Born in Angers, France, in December 1944. Originally trained as an Engineer in Aeronautics, turned to more theoretical problems in mathematical physics, applied mathematics and theoretical mechanics while obtaining his M.A. and Ph.D. at Princeton University (1971) as a NASA Fellow and his Doctorat d'Etat ès Sciences Mathématiques (Paris, 1975). Directeur de recherche au C.N.R.S.; part time UNESCO Visiting Professor at the University of Cairo. Has been teaching at the Université Pierre et Marie Curie (Paris) since 1972. Visiting Professor at Princeton (1975-76, 1978), Istanbul (1980), Stockholm (1984); Genoa (1984) Nottingham (1985) and Pisa (1990). Present interests: Nonlinear phenomena, more particularly solitons in solid state physics and biological systems, and the theory of fracture. — Address: Université Pierre-et-Marie Curie, Lab. Modélisation en Mécanique, Tour 66, 4 Place Jussieu, F-75252 Paris Cédex 05.

What they call an Introduction

Turned by formation and experience toward the other side of the Atlantic Ocean but sentimentally attached to eastern Europe, I find in my stay in Berlin a true *recentrage* (as they say in French politics) and a necessary reshaping of scientific co-operation on the European scene. For an active scientist who can neither forget his international scientific environment nor neglect his daily contacts with students and colleagues, a ten-month stay at the Wissenschaftskolleg may look like a parenthetical episode; but an extremely busy one indeed, one that will remain as a most fruitful and, therefore, enjoyable one. During that almost one-year span I used the opportunity to put together the pieces of a scientific puzzle and to deepen and unify other aspects while pursuing works in collaboration with co-workers harmoniously spread over the world.

A true puzzle

It has been remarked upon by some psychologist that a common feature of creative life is the development of a "network of enterprises", projects and lines of thought that to the outsider might seem unrelated. May I confess that this lack of sensitivity to a deeply rooted relation is also shared by the scientist who is not himself aware of the underlying unity to which he contributes. Let me illustrate this. Previous to my stay in Berlin I had been very active in quite distinct fields of applied mathematics and physics including the electrodynamics of continua (dear to Maxwell and Einstein), the theory of plasticity and fracture (an engineering touch with real conceptual and mathematical difficulties) and nonlinear-wave propagation in solids (the dessert for the true *afficionados).* Strangely enough, three volumes relating to these three subjects and synthesizing my own and others' research works were published in the original or in translation during this year [1–31. In recent years, however, I have been puzzled by the notion *of pseudomomentum* or *crystal momentum* whose role in solid state physics was emphasized by Sir Rudolph Peierls and by the notion of *"configuration force"* introduced by J. D. Eshelby (now deceased) in 1951 while dealing with the field theory of defects and singularities (in the mathematical sense) in elastic bodies. 1 had decided to devote my stay in Berlin to thinking the matter over and, if feasible, writing a book on the subject. But what is this about? It is clear that tearing a sheet of paper shows that the *crack* (fracture) thus produced is developing *in* the paper and *not* in the physical space in which the said sheet is embedded; no physical force of the Newtonian type is acting at the tip of the progressing crack as the only physical forces applied are those transmitted through the hands holding the paper. However, a kind of negative photographic view of this picture allows one to imagine that a *fictitious force* is acting *on the material manifold* (space) and is *sucking in* the crack, thus favoring its thermodynamically irreversible expansion (in the same way as a true Newtonian force acting at the end of a plastic wire favors its elongation if the said force is intense enough).

The theory of *material forces on material space* is what we have developed to cope with the above-described phenomena. It allows one to effectively compute those *fictitious* forces from the solution of the mechanical problem at each instant of time. Then this "force" is used as a *criterion* to decide on whether the crack will progress or not. This is contemplated in so-called quasi-statics. In dynamics, however, the associated *material balance law* (i.e. fundamental expression of a physical law) is none other than the *conservation* or *nonconservation* (in the presence of material inhomogeneities) of the above-mentioned pseudomomentum. Moreover, in the case of *electrodynamics,* the rational construction of this equation provides an unequivocal means to effectively define the notion of *electromagnetic momentum* in magnetized and electrically polarized *matter* by discriminating automatically those typically *material* entities from the pervasive pure electromagnetic-field ones. This seems to shed light on the old unresolved controversy of electrodynamics in matter (going back to Minkowski, Abraham and Einstein): it all depends on whether you think in physical or material space! Furthermore, depending on the degree of mathematical singularity of the deformation field of material space, the latter may present a more or less complex, non-Euclidean, or even non-Riemannian, intrinsic geometry, somewhat like the space-time structures envisaged in the most advanced theories of unifying forces. Finally, it was shown that *solitons* (types of nonlinear waves enjoying particularly nice properties during interaction between one another — *intercourse* would not be correct although a linear-like superimposition (solitonic property) prevails giving birth to a phase shift only!) propagating in elastic matter do behave as *material* particles on the corresponding material manifold (i.e. they check the global balance of *pseudomomentum!).* This trifling remark provides in fact an operational tool to study the loss of solitonic property under the influence of external stimuli or defects by looking at the perturbation in the then true un-balance of pseudomomentum.

We see that the law of balance or un-balance of pseudomomentum (like the un-birthdays of Lewis Carroll, un-balances are much more frequent, interesting and fun than pure balance laws) provides the connecting thread between fracture, electrodynamics, crystal physics and soliton theory. This view is expanded in a book in press for which we have coined the expressions *"Eshelbian mechanics on material manifolds"* and *"Theory of material inhomogeneities"* [4].

Warum ist die Null so schön, Professor Maugin?

The Berlin of the dreams of my adolescence remains the one of the *Kaiser Wilhelm Institut fill. Physik* haunted by M. Planck and A. Einstein, and of *Der blaue Engel* of Marlene Dietrich; quanta, relativity and *UFA,* a strange combination indeed. I had no opportunity to frequent music-hall artists and, like Professor Unrat, switch to the mortifications of cheap show business. But a stay at the Wissenschaftskolleg seems to bring original and surprising elements with which a "normal" scientist is not familiar. In our profession, we are not supposed to be heard of or written about.

However, the WIKO-label helping, I found myself being photographed in Rodin's thinker-like pose in front of the *Einsteinium* in Potsdam Observatory, interviewed on my *Weltanschauung* in a usually more serious German *Zeitung*, taken as some kind of model for, I am afraid, a mad scientist by a successful Berlin novelist, commented upon by a world literary celebrity in a foreign magazine, brought some message by a fetish actor from the film director Wim Wenders, and trying to preach the gospel of (reasonable) *nonlinear science* to a bunch of nice, but not always reasonable, humanists, i.e., my fellow colleagues at the Kolleg. Perhaps it was some virus which caught at the Wissenschaftskolleg itself and the privilege of having been the only "hard" scientist in residence during this memorable year?

Meeting other professional scientists

I gave lectures in Germany at the Technische Universität Berlin, Universität Stuttgart, Technische Universität Darmstadt, Universität Paderborn, Humboldt Universität Berlin, Universität Potsdam, Universität Leipzig, and abroad in Paris, Messina, Palermo, Catania, Poznan, Dublin and Florida. A joint Workshop of the Wissenschaftskolleg and the Technische Universität on "Nonlinear Thermodynamical Processes in Continua" (11— 12 June 1992) took place at the TU Berlin with the efficient cooperation of W. Muschik. It gathered seventeen renowned lecturers from Germany, France, Italy, Poland, Egypt, Canada and the U.S.A. and a much larger audience. The proceedings of this extremely dense and exciting meeting shall shortly be published in book form [5].

I wonder why they call these *sabbatical* years!

References

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2 G. A. M., *The Thermomechanics of Plasticity and Fracture,* Cambridge University Press (Textbooks on Applied Mathematics), U.K., (First Edition, May 1992) — Russian and Japanese translations in the process.

3 G. A. M., et al., *Nonlinear Electromechanical Couplings, J.* Wiley and Sons (Nonlinear Science Series), New York, First Edition, 1992.

4 G. A. M., *Material Inhomogeneities in Elasticity,* Chapman and Hall Series in Applied Mathematics and Mathematical Computation, London (in the press, 1992).

5 W. Muschik and G. A. M. (editors), *Nonlinear Thermodynamical Processes in Continua,* Teubner-Verlag, Leipzig und Stuttgart (in the press, 1992).