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Localization Phenomena; Asymptotic Properties

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During the academic year 1990/91, I have participated in the annual project *"Geometrie and Physik"* organized by the *Wissenschaftskolleg zu Berlin.* Among the many problems I have discussed with other participants of the project, I would like to single out two main topics: (1) Localization phenomena in multi-dimensional quasi-periodic media; (2) Asymptotic properties of Lyapunov exponents for one-dimensional chaotic media in a state of weak disorder.

(1) The first direction of my research is related to previous work by J. Bellissard et al., by Ya. G. Sinai, and by J. Frohlich, T. Spencer, and P. Wittwer. The approaches used by the above mentioned authors have both common features and substantial distinctions. The first of them developed by J. Belissard, R. Lima, and E. Scoppola, is based on a linear version of the famous KAM (Kolmogorov-Arnold-Moser) techniques and suggests a very convenient language of C*-algebras. This approach has been used for investigation of spectral properties of a class of quantum Hamiltonians describing incommensurate lattices of any dimension. However, this approach, in its original form, does not allow the investigation of the tunneling phenomena in more general incommensurate media. Two different approaches have been developed to analyze localization phenomena in quasi-periodic media with infinitely many resonances. One of them, proposed by Ya. G. Sinai, provides rich information about the en-

ergy spectrum and eigenfunctions of the Hamiltonians describing the quasi-periodic media. The second one, proposed by J. Frolich, T. Spencer, and P. Wittwer, is based on a detailed analysis of the Green's functions corresponding to the quantum Hamiltonians in incommensurate lattices. However, both of the approaches originally were applied to one-dimensional lattices. It turned out that combining the advantages of all of the above mentioned approaches, one can investigate a far more general class of incommensurate media. A part of this program has been implemented in our joint work with E. I. Dinaburg, also a participant of the annual project. We have proved that the localization phenomenon takes place for a wide class of long-range multi-dimensional quasi-periodic operators with one basic frequency. A different class of short-range multi-dimensional quasi-periodic operators with two basic frequencies has been studied by T. Spencer (a short-term visitor of the Kolleg within the framework of the annual project) and myself. The paper is now in preparation, and a very stimulating atmosphere, typical for the Wissenschaftskolleg, as well as valuable opportunities to discuss the problem with a number of our colleagues, made an essential contribution to that work.

(2) The asymptotic properties of Lyapunov exponents in a state of weak disorder have been a challenging problem for many years. Numerous physical, computational, and rigorous mathematical papers have been devoted to this problem. One of the "hard nuts" in this field was the class of disordered media generated by smooth finite-dimensional chaotic dynamical systems. Surprisingly, even in the case where the underlying dynamical system possesses very strong ergodic properties (such as exponential mixing, finite Markov partitions etc.), most of the modern and classical methods cannot prove positivity of Lyapunov exponents for relevant disordered media. T. Spencer and I had discussed this problem earlier, but crucial progress has been made during the short visit by T. Spencer at the Wissenschaftskolleg. The problem has turned out to have a simple solution, and now we are extending the method we have developed for strongly mixing media to a wider class where the mixing properties can be essentially relaxed. This seems to be not the final, but rather a starting point for further work.

Although this is a report on the research carried out within the framework of the annual project in 1990/1, the influence of the project on my research is not limited to the papers written or prepared to publication during that period. It has contributed to my plans of future work and allowed me to develop new points of view regarding the problems I had studied before.