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Material-Evolution

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The studies carried out at the Wissenschaftskolleg were mainly directed towards the formulation of *a general theory of evolution, structural change and stability of heterogeneous materials under the effect of external thermo-mechanical influences.* It is evident that such a theory is important for the understanding and analysis of the response behaviour of various discrete media, which in general also depends on the degree of interactions between the constituents comprising the microstructure. It is known that there exists a great variety of discrete media ranging from high-temperature resistant compounds to materials encountered in biophysical/chemical applications. Due to the inherent randomness of the physical and geometric properties of the material structure a general *random theory of deformation* has been developed in previous work. It has been shown in [1] that a rigorous formulation of the response behaviour on the basis of the mathematical theory of probability and the axioms of measure theory is possible by considering the relevant field quantities as random variables or functions of such variables. Hence the ensuing stochastic analysis has been carried out by conveniently adopting an operational formalism in which the physical system is represented by an *abstract dynamical* one. The latter is usually characterized by the triple *[X, .,:P*]. where *X* is a probabilistic function space, t the o algebra of Borel sets (events) and 9 an appropriate probability measure on the subsets of *X*. Evidently the response of the medium at a given instant of time corresponds to the state of the material at this time and can be described by a set *of state vectors* belonging to a subspace Z (state-space) of the probabilistic function space *X*. The state vector i EZ c *X; (a = 1,.. . N)* of an individual element of the microstructure has generally several components according to the set of *internal variables* $\{x_i\}$, $(i = 1...n)$. It is apparent that in the *stochastic state-space* representation of the material behaviour, the state-space Z and any of the chosen subspaces must be given a suitable topological structure and an appropriate measure. The evolution of states of the medium is then characterized by a stochastic process $z_t \in Z$. A distinction must be made however between the *observable* and *unobservable variables*, i.e. the components of the process z_t. This becomes particularly important when transients are considered in which structural changes occur. It has been shown [2, 3] that a particular state zof the medium during this period can be specified by a *stochastic system function.* The latter is in fact a vector functional containing in its argument the variables $z_t \in Z$, a continuous or a discrete time set $t = 0, 1, ... \in T$ in accordance with the observations at these times and p_t certain control parameters that are responsible for the structural change during the evolution of the material system. The control parameters belong to a control space $\Omega \subset Z$. This approach leads then to a formulation of the structural change and the transient behaviour in terms of *a probabilistic state function* F_r for an arbitrary time instant $r \in T$ from a given initial state z_{t0}. The probabilistic state function F_r is assumed to beg, x g_T measurable, where $_{IT}$ is the a-algebra of the control variables p_r and ?^{r} an appropriate measure. The evolution of the state process z_t can be conveniently described by a partially observed jump Markov process, which has two main characteristics, i.e. the jump rate and the state jump distribution. Since the latter depends on the current value of the *unobserved components* of z, and the history of observations up to the time instant r, it becomes necessary to consider an increasing family of sub-a-fields of the algebra *gz on* Z. Thus, ifs$_r$z, *t* E Tdenotes the subalgebra for a continuous time set: $t_0 < t$, *!;, ⊂ J* and for a discrete time set: $t = 0, 1, 2 ... \in T$ with $t_0 < t_1 < t_2 ...$; *gf*$_0$ $_2$..., the family *{. }* is a *filtration* of the space [Z, *gz]* and describes the history of *z,* on Z. Hence a real-valued stochastic process z,or a sequence of random variables { z,} will be a martingale on the stochastic dynamical system [Z, Wiz, *r]* with filtration { } satisfying certain conditions [3].

Although the system theoretical approach to the evolution and structural changes of a discrete medium permits a description in terms of Markov step processes certain *internal mechanisms* within and between elements of the structure influence significantly the relevant field quantities. There is a great variety of such mechanisms and interaction effects, which are of a particular form for the given medium under consideration. They may also be regarded as *unobservable variables* in addition to those contained in the state vector. These additional random variables effect disturbances from local equilibrium states of the structure. In general a stochastic dynamical system with random disturbances can be described by a non-linear stochastic differential equation. If the disturbances are small so that only slight deviations from the equilibrium state are induced, the system can be regarded as essentially linear in state and the disturbances equivalent to a white Gaussian noise process. The phenomenological representation results then in a stochastic differential equation of the Langevin type. By considering the n-dimensional state vector and its evolution through time, the white Gaussian noise process, i.e. $\{Q, t, t_o\}$ can be replaced by the formal derivative of the Brownian motion process {ß,} so that an appropriate stochastic differential equation is obtained. the solution of such an equation must be interpreted however in the sense of Itô's integral equation (see also [3]). On the assumption that at z_b, i.e. the beginning of the evolution of the state process z,is independent of the Brownian motion process, the solution of the above differential equation will be a strong Markov process even a diffusion process. Hence the evolution is then determined by the probability density function of the relevant field variables. To include the sequence of observations or measurements in the formulation of the response behaviour requires the use of the conditional probability density as discussed in [3]. In conclusion, it is to be noted that so long as independence of the relevant field variables exists, the evolution of states and a structural change of the medium will be *subcritical.* In the case of critical behaviour of the material, where phase-transitions occur, the field variables become strongly dependent. Hence the collective behaviour of the medium will then be represented by strongly correlated quantities for the individual elements of the structure resulting in limit-measure that cannot be considered anymore as product measures. A more comprehensive analysis of the evolution, structural change and stability of discrete media is given in chapts. 3 and 4 of reference [4].

References

1 Axelrad, D. R., *Foundations of the probabilistic mechanics of discrete media,* Monograph, Oxford: Pergamon Press, 1984.

2 Axelrad, D. R., "The transient behaviour of structural solids", In: *Lecture Notes in Physics,* No. 249. Eds. E. Kröner and K. Kirchgässner. Berlin: Springer Verlag, 1985.

3 Axelrad, D. R., *Stochastic analysis of structural changes in solids,* Seminar at the Wissenschaftskolleg 23.-24. Febr. 1987, Tagungsband, Berlin-Heidelberg: Springer-Verlag, 1987.

4 Axelrad, D. R., *Evolution, change and stability of microstructures,* Monograph, Oxford: Pergamon Press, forthcoming.