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The Choice Axiom of Luce and Stochastic Social Decisions

Diese Arbeit untersucht die Überführung (Aggregation) stochastischer individueller Auswahlfunktionen in eine stochastische soziale Auswahlfunktion. Es wird angenommen, daß soziale Auswahlfunktionen das Auswahlaxiom von Luce und eine Regularitätsbedingung erfüllen und daß die Menge aller Auswahlfunktionen, die diesen beiden Forderungen genügt, die zulässige Menge individueller Auswahlfunktionen definiert. Weiterhin wird postuliert, daß die soziale Entscheidungsregel die probabilistischen Gegenstücke der Arrow'schen Bedingung der Unabhängigkeit irrelevanter Alternativen und des Einstimmigkeitskriteriums erfüllt. Unter diesen Voraussetzungen kann die Existenz einer Person mit folgender Eigenschaft nachgewiesen werden: Für jede durchführbare Menge von Alternativen sind die von diesem Individuum den einzelnen Alternativen zugeordneten Auswahlwahrscheinlichkeiten gleichzeitig die entsprechenden sozialen Auswahlwahrscheinlichkeiten, und zwar unabhängig von den Auswahlwahrscheinlichkeiten anderer Individuen.

1. Introduction

The purpose of this paper is to develop a model of group choice where the choice functions of the individuals constituting the group, as well as the choice function of the group itself, are assumed to be stochastic in nature. Assuming that all these choice functions satisfy the Choice Axiom of Luce [13, 14] and the assumption of >regularity<', we investigate the structure of group decision functions (i. e. procedures for aggregating the choice functions of individuals into a group choice function) which satisfy the stochastic counterparts of Arrow's independence of irrelevant alternatives, and the unanimity criterion. Since the literature on collective choice theory has mostly concentrated on deterministic choice models[2], and since many of the existing contributions which do investigate the problem of group choice in a stochastic framework, use assumptions different from ours, it may be worthwhile spelling out in some detail, our motivation for: (a) using a stochastic framework, and (b) using a set of assumptions different from those figuring in many earlier contributions which also develop stochastic models of social choice.

In the recent past, psychologists have increasingly modelled individual choice behaviour in stochastic terms. Surveying this literature nearly two decades ago, Edwards [8] wrote, »In 1954 the theories of choice were mostly deterministic ... The major recent theoretical development is a shift from deterministic to stochastic models«. Two decades later now, the transformation seems to be almost complete, and stochastic individual choice models seem to be the rule rather than the exception in psychology.[3] The significance of this for the theory of collective choice is obvious. If in certain contexts, individual choice behaviour is assumed to be stochastic, then clearly in the normative theory of collective choice, it will be natural to assume that social choice is also stochastic in those contexts. If given two alternatives, x and y, every individual in the society chooses x with probability .7 and y with probability .3, then it is not at all obvious why one should require social choice over {x, y} to be deterministic. Independently of this consideration, the introduction of a stochastic group choice function has also been often motivated by the possibility that it might provide an escape route from the classic impossibility result of Arrow in so far as it opens up another dimension over which reconciliation of conflicting preferences could be achieved. Thus if there are two alternatives x and y and three individuals two of whom prefer x and one of whom prefers y, the probabilistic social choice model does not force us to say that given {x, y}, society must choose x alone ory alone (or alternatively, that it does not matter which alternative is chosen); the probabilistic framework allows a wider range of possibilities by permitting non-trivial social choice probabilities for x and y.

These are some of the reasons why it may be of interest to construct models of social choice where social and individual choice functions are assumed to be stochastic. However, in constructing such models one clearly needs simple and intuitively plausible axioms governing the probabilistic choice functions — axioms which will perform the role of the >rationality< postulate in traditional economic theory. Many of the earlier contributions assumed the probabilistic choice functions to be rationalizable in terms of random orderings (see [2], [3], [11], and [19]).[4] In contrast, we have chosen to assume the Choice Axiom of Luce and the property of regularity, for the probabilistic choice functions. This is because of several reasons. First, in terms of experimental evidence, rationalizability in terms of stochastic orderings does not seem to have fared much better than Luce's Choice Axiom or regularity, as a descriptive hypothesis about individual behaviour. At the same time, the assumption of rationalizability in terms of stochastic orderings does not seem to have any compelling appeal as an intuitively transparent canon of probabilistic rationality. Given this, it seems desirable, also to explore the implications of alternative assumptions such as Luce's Choice Axiom and regularity, which have considerable intuitive appeal as properties of rational probabilistic behaviour (we take up this point again in Section 2).

In our main results we assume that social choice functions satisfy the Choice Axiom of Luce and regularity, and that the set of all choice functions satisfying Luce's Choice Axiom and regularity constitutes the permissible set of individual choice functions. In addition to this, the group decision procedure is assumed to satisfy the probabilistic counterparts of Arrow's independence of irrelevant alternatives, and the unanimity criterion. Given these assumptions we demonstrate the existence of an individual such that for every feasible set of alternatives, the choice probabilities attached by him to the different alternatives emerge as the corresponding social choice probabilities irrespective of other individuals' choice probabilities. Thus given our assumptions, the permissible group decision procedures degenerate into trivial dictatorial forms (reminiscent of dictatorship in the deterministic structure of Arrow) despite the wider stochastic framework. This dictatorship result is much stronger than the central propositions of several earlier contributions (see, for example, [2], [3], [4], [10], and [19]) which essentially established a subadditive power structure.[5] It is also worth noting that our conclusion regarding the power structure applies to all feasible sets of alternatives, irrespective of their size; this is in contrast to the conclusions of the contributions cited above which derive restrictions on the structure of power with respect to two-element feasible sets but not feasible sets with more than two elements.

2. The Notation and Definitions

N= { 1, 2,...,n) is the set of all individuals constituting the society and *X* is the set of alternatives (œ > *n* > 2 and c > IX I > 3). The set of all non-empty subsets of Xis denoted by 2'; the elements of *2'* are called *issues.*

Definition 2.1: A probabilistic choicefunction (PCF) is a function *p: Xx2'* [0,1] such that for all *A e 2',* $\sum_{x \in A}$ *p(x, A) =1* and for all xE *X-A, p(x, A) = 0.*

Thus given any feasible set *A* of alternatives, a PCF specifies the probability of choosing the various alternatives. When an issue has just two elements - say x and y, to lighten the notation we write simply *p(x,* y) and *p(y, x)* instead ofp(x, {x, y)) and *p(y, {x, y})* respectively. Similarly, *p(x, x)* = *p(x, {x}) =1.* Given a PCFp and given two issues *A* and *B*such that *B ç A, p(B, A)* stands for $\sum_{x \in B}$ *p(x, A).* The set of all PCFs will be indicated by *P.* For all *p* e *P,* and for all *A* E *X, A' (p)* stands for {xe *Al p(x, y)* > O for ally *A}.*

Definition 2.2: A probabilistic group decision function (PGDF) is a function *f:* P"-- *P* where 0# P *c P.*

The n-tuples of PCFs in the domain of a PGDF are indicated by *q* = *(q1,. • .,q"), q' = (qi,• • .,q ")* etc. and we write *p = f(q), p' = f(q')* etc. q_i is to be interpreted as the PCF of the i-th individual while *p,p'* etc. refer to

PCFs of the society. The domain and the counterdomain of a PGDFfwill be indicated by D_f and CD_f respectively.

Definition 2.3: Let *f* be a probabilistic group decision function. *f* satisfies

(2.3.1) *Binary Independence of Irrelevant Alternatives* (BIIA) iff for all *x, y,* EX, and for all *q, q'E* D_f, if *[$q_;$(x,y)=qj(x,y)* for all ie *N],* then *p(x, y)= (x; Y);*

(2.3.2) *Binary Unanimity Rule* iff for all x, ye *X,* all qc D_f, and for all to [0,1], *[$q_;$(x, y) > t* for all ie *N]* implies *[p(x, y) > t];*

(2.3.3) *Binary Neutrality* (BN) iff for all *x, y, x', y'* E *X,* and for all *q, q' E* D_f, if (for all *i E N, $q_;$(x, y) = q (x', y')),* then *p(x, y) = p'(x',* y').

(2.3.4) *Binary Monotonicity* (BM) iff for all *x, y* e *X,* and for all *q, q' E* D_f, if (for all i E *N q;(x, y)> $q_;$(x, y)),* then *p'(x, y) z p(x, y).*

(2.3.5) *Binary Anonymity* (BA) iff for all x, yE *X,* all *q, q' e* D_f, and every one-to-one function *M* from *Nto N, ($q_;$(x, y) = q_{mo} (x, y))* implies *(p(x, y) =p'(x Y))*•

Most of these properties are probabilistic counterparts of corresponding well known properties in the deterministic framework, and therefore hardly need any comment.

So far we have not introduced any restriction on the domain or counterdomain of the PCDFf These restrictions will clearly involve assumptions about what we consider to be the set of permissible individual PCFs and what we consider to be suitable >rationality< properties to be imposed on social PCFs. To specify our restrictions on D_f and CD_{fi} we first introduce certain properties of a PCF. Let 9P be the set of all linear orderings over *X.*

Definition 2.4: Let *p* be a PCF. *p* satisfies

(2.4.1) *Rationalizability in Terms of Stochastic Orderings* (RSO) iff there exists a function *e:* -- [0,1] such that: (i) for all RE M, *e(R) >* 0, and $_R$E$_D$ *e(R) =1;* and (2) for all xe X and all *A e 2', p(x, A) =*$_R$*E e(R)* where

92 = {$_{RE}$.*9 I x* is the R-greatest element in A};

(2.4.2) Luce's Choice Axiom (LCA) iff for all *A,B,CE2'* such that *CcBcA,* if *[p(x,y)0* 0 for all x, *yEA],* then *,g(C,A)_E(C,B)•E(B,A);*

(2.4.3) *Regularity* (Reg.) iff for all *A,* Be 2' such that *Bc A, [p(x, A) < p(x, B)* for all *xe B];*

(2.4.4) Strong Stochastic Transitivity (SST) iff for all *x, y,* ZE *X, (p(x, y) > .5 & p(y, z) > .5)* implies *[p(x, z) > max(p(x, y), p(y, z))].*

Many earlier writers have imposed the property of RSO on social PCFs (see [3], [2], [11] and [19]) and on individual PCFs (see [11]). Intuitively, RSO implies that the probabilistic choice function under consideration could have been >induced< by a lottery over possible orderings. We do not postulate RSO either for individual PCFs or for social PCFs. In our main result we assume that $D_f = L''$ and CD_f c L where f is the PGDF and L is the set of all PCFs satisfying LCA and Reg. What we call LCA is really only a part of the Choice Axiom as originally introduced by Luce [13]; we have not included the other part since it follows directly from regularity which we assume for social and individual PCFs. Reg. requires that the choice probability for an alternative should not increase as we go from a smaller issue to a larger issue which includes the smaller one. This seems an intuitively compelling assumption to make for social as well as individual PCFs. LCA applies non-trivially only when none of the pairwise comparisons is perfect. As has been often noted in the literature (see Luce ([13], [14])), LCA embodies the probabilistic version of the principle of independence of irrelevant alternatives, which, in various forms, figures in Chernoff [5], Nash [16], and Radner and Marschak [17];[6] and as such it has a powerful intuitive appeal as a normative principle of stochastic rationality that may be imposed on social PCFs. The case for postulating LCA for individual PCFs is less clear. As a descriptive hypothesis about individual choice behaviour, the limitations of LCA are well known. Several examples and experimental results suggest that as a descriptive postulate about individual behaviour, LCA may not be appropriate in many contexts.? However, this is also true of most other properties (including RSO) often postulated for the probabilistic choice behaviour of individuals. As Luce [14] observes in his lucid and balanced survey, »... once we enter the path of strict rejection of models on the basis of statistically significant differences, little remains. To the best of my knowledge, the only property of general choice probabilities that has never been empirically disconfirmed is regularity ...« As against this inadequate empirical support for most properties, including RSO and LCA, postulated for probabilistic individual choice behaviour, there is the need for simple assumptions about >rational<, stochastic, individual choice, which can constitute the basis of theoretical models. It is this consideration, rather than considerations of universal empirical confirmation, which, at the present stage of our knowledge, lends interest to analysis based on the assumption of LCA (or alternatively, on the assumption of RSO) for individual PCFs.

3. Some Implications of Luce's Choice Axiom and Regularity

In this section we explore certain implications of LCA and Reg., which we later assume for social as well as individual PCFs.

Proposition 3.1: Let *p* be a PCF satisfying LCA, and *A E* 2' be such that A = *A+ (p)* (i. e. within *A* no pairwise discriminations are perfect). Then

(3.1.1) for all BE ' such that *Bc A, [p(x, A) < p(x, B)* for all xe *B].*

(3.1.2 Luce [13]) for all *x, y, zEA, [p(x, y) > .5 & p(y, z) > .5]* implies *[p(x, z) z* ^{max} *(p(x, y),* $^{p(y,}$ *z))];*

(3.1.3) for all x, y, ZE *A,* if *p(x, y) = 0* and *p(x, z) = 0 ',* then *p(z, y) = (1-0* ')/(1-28' + *[B'/0]);* in particular, when *p(x, y) = p(x, z) = 0 , p(z, y)=* ^{1}*12;*

(3.1.4) for all *x, y* E *A, p(x,A)/p(y,A) = p(x,y)/p(y,x).*

Proof: The reader may refer to Luce [13] for the proofs of Propositions 3.1.2 and 3.1.4. Luce [13, p. 16] also shows that given *A = A+ (p),* for all x, *y, ze A, p(x, y) • p(y, z)- p(z, x) = p(x, z) • p(z, y) • p(y, x).* With straightforward manipulation, this leads to Proposition 3.1.3. Proposition 3.1.1 follows immediately from LCA, since by LCA, for *all xe Bc A, p(x, A) = p(x, B). p(B, A)* and since 0 < *p(B, A) < 1.* •

Propositions 3.1.1 and 3.1.2 show that in the special case where *p(x, y)* *0 for all x and y belonging to the issue *A* under consideration, LCA implies Reg. and SST over *A.* Therefore the use of Reg. (or SST) besides LCA imposes additional restrictions only when *p(x, y)* =0 for some *x* and y belonging to the relevant issue.

Proposition 3.2: Let *p* be a PCF satisfying Reg. Then

(3.2.1) for all *A* E2' and for all xE *X,* if *[xEA&p(x,y)=*0 for some yE *A],* then for all BE .r such that *Bc A, p(B, A) = p(B-{x}, A-{x});*

and (3.2.2) for all x, y, z E X, if *[p(x, y)* =1 or *p(y, z)=1],* then *[p(x, z) . min (P(x, Y), P(y, z))1*

Proof: (3.2.1) Consider any xE *A* such that *p(x, y)* = 0 for some *ye A.* Then by Reg., *p(x, A)* = 0 and hence $\sum_{ZE(A-\{ \})}$ *p(z, A)* = 1. Again by Reg., for all *wE (A-{x}), p(w, A-{x}) > p(w, A).* Since $\sum_{Ze (A-\{x\})}$ *p(z, A-{x}) = 1 =* $\sum_{ZE (A-\{x))}$ *p(z, A),* it follows that for all z€ *(A-{x}), p(z, A-{x}) = p(z, A).* Hence *p(B, A) =* $\sum_{ZEB}$ *p(z, A) =* $\sum_{ZE (B-\{XD}$ *p(z, A) =* $\sum_{ZE (B-\{x\})}$ *p(z, A-{x}) = p(B-{x}, A-{x}).*

(3.2.2) Without loss of generality assume that $p(x, y) = 1$. Then *min* $(p(x, y), p(y, z)) = p(y, z)$. Suppose $p(x, z) < p(y, z)$. Then we show a contradiction. Let $A = \{x, y, z\}$. By Reg., $p(x, A) < p(x, z)$ and $p(z, A) < p(z, y)$. Hence $p(x, A) + p(z, A) < p(x, z) + p(z, y) < p(y, z) + p(z, y) = 1$. However, given that $p(x, y) = 1$, by Reg. we have p(y, A) = 0. Therefore $p(x, A) + p(z, A) = 1$, which is a contradiction. ■

Proposition 3.3: Let *p* be a PCF satisfying LCA and Reg. Then

(3.3.1) for all *0* e].5,1 [and all x,y,ze *X*, if[p(x,y) z .5& $p(y,z)=0]$ or $[p(x, y) = B$ & $p(y, z) >$ .5], then $p(x, z)$ z B .

(3.3.2) for all x, y, ze *X*, if $p(x, y) = p(y, z) = 1$, then $p(x, z) = 1$.

Proof: (3.3.1) If $p(x, y) = 1$ or $p(y, z) = 1$, the conclusion follows immediately from Proposition 3.2.2. On the other hand, if $p(x, z) = 1$ the conclusion is trivial. Suppose therefore $p(x, y) < 1$, $p(y, z) < 1$, and $p(x, z) < 1$. To show $p(x, z) > 0$, suppose on the contrary $p(z, x) = 1$. Then again by Proposition 3.2.2, either $p(y, x) > e$ or $p(z y) > e$, establishing a contradiction. So $p(x, z) > 0$ and Proposition 3.1.2 is applicable to x, y and *z* with $A = \{x, y, z\}$, completing the proof.

(3.3.2) The result follows immediately from Proposition 3.2.2. ■

4. The Main Results

We now explore the structure of PGDFs satisfying BILA and BUR when social and also individual PCFs are assumed to satisfy LCA and Reg. We first prove the following proposition.

Proposition 4.1: Let *f: L" - L* be a PGDF satisfying BIIA and BUR, where *L* is the set of all PCFs satisfying LCA and Reg. Then (satisfies BN and BM.

Proof: First we show that

for all distinct *a, b,* CE Xand all *q, q' c* D_f, if *q;(a, c) > q,(a, b)* for all *iEN,* then *p'(a, c)* *p(a, b)* ... (4.1)

Consider *q'e* D_f such that for all *le Nq;(a, b) = q,(a, b) & q;{a, c) = q;(a, c) & q;{b, c) = 1*. (It can be checked that given $D_f = L'$ and *q;{a, c) > q,(a, b)* for all *i e N,* such a *q 'e* D_i can be found. Note that for all *le N,* LCA is trivially satisfied by *q)* Comparing *q* and q"and using BIIA we have *p'(a, b) = p(a, b)*. By BUR, *p'(b, c)* = 1. Given that pr'satisfies Reg., by Proposition 3.2.2, *p"(a, c) > p'(a, b)*. Hence by BIIA,p'(a, *c) = p"(a, c)>. p(a, b) = p(a, b).* This proves (4.1).

It follows from (4.1) that for all distinct *a, b,* ce *X* and all *q, q' E* D_f, if *q;*

(a, c) = q; (a, b) for all *le N*, then *p'(a, c) = p (a, b)*, and noting that for any PCF *r*, and any *â, b e X, ,(â, b) + r(b, 6)* = 1, if *q',(c,* a) = *q,(b,* a) for all i e *N*, then *p'(c, a) = p (b, a)*. BN now follows using the Arrow technique (1963, pp 99-100), and BM may be deduced from 4.1 together with BN. •

Remark 4.1: It maybe noted that the proof of Proposition 4.1 does not require social PCFs to have the property LCA. Moreover, it is not difficult to modify the construction of e(used to derive (4.1)) so that the proof is valid when individual PCFs satisfy LCA, Reg. and SST, provided social PCFs satisfy at least SST. It will also be valid when D_f is less or CD_f more restricted.

Remark 4.2: If L figuring in the statement of Proposition (4.1) is the set of all PCFs satisfying either LCA and Reg., or LCA, Reg. and SST, then general properties of neutrality and monotonicity (involving issues with arbitrary numbers of alternatives) can be proved.

Theorem 4.1: Let f L"-L be a PGDF *which satisfies* BIIA *and* BUR, *L being the set of all* PCFs *satisfying* LCA *and* Reg. *Then there exists a unique heN such that for all geD*$_f$*, p(q) =* q_h.

Proof: We first show that

there exists *h* e *N* such that for all *a, b e X* and for all 4 e *L"*, if 4_h *(a,b)* = . 5, then *p (a, b) > .5*. ... (4.2)

Let x,y and *z* be three distinct alternatives. Let *N** be a smallest subset *N'* of *Nsuch* that for all *4E L"*, if 4;(x, y) = .5 for all *ie N'*, and 4_1(y, x)= 1 for all *je N-N'*, then ß(x, y) _ .5. Given BUR, such subsets exist and are nonempty. Let *he N**. Construct *q* e *L"* such that [for all ie *N*-{ h), q;(x, y)= q,(x,z)=* .5 & *q;(y,z)=* 1]; [for all je *N-N*,q_i(y, x)=g_1(z, x)=g_1(y,z)=1]*, and *[q_h(x, y) = q_h(z, y)* = .5 & *q_h(z, x) = 1]*. It can be checked that such a *qeL"* exists.

By BUR, *p (x, z) <* .5, and given BIIA and BN (which follows by Proposition 4.1), *[p (x, z)* =.5] contradicts the specification of *N**. Hence *p (z, x)* > .5. Again by the specification of *N*, p (x, y)* = .5. Applying Propositions 3.2.2 and 3.3.1, *p(z, y) z .5*. (4.2) now follows by BN and BM.

Suppose next *0* e] .5,1[. Construct *q' e L"* such that [for all *ie N-{14 , q'; (y,x)=q;(z,x)=* 1 *&q'r(y,z)=* 0] and *[q'(xy)=.5&gh(x,z)=gh(v,z)= 0]*. By (4.2), *p'(x, y) >* .5 and by BUR, p'(y, *z) = 0* . Applying Proposition 3.3.1, *p'(x, z) z 0.* It then follows by (4.2), BN and BM that

for all *0* e [.5, 1[, for all a, be *X* and for all *4e L"*, if $_h$*(a, b) = 0* , then *0* ... (4.3)

Now consider any *a, b* e *X* and *4* e *L"* such that q_h *(a, b)*=1. Let 8 *c[.5,1 [* and let *q'e L"* be such that *q',,(a, b) = 0* and *[q~ (a, b) = 4, (a, b)* for all *i e N*–

(b}]. By (4.3), p'(a, *b) > 0* and by BM, *p(a, b) z pta, b) > 0* . Since this is so for all *0* E [.5, 1[, *p(a, b) = 1*. (4.3) may therefore be replaced by

for all *0* E [.5,1], for all *a, b,* E *X* and for all *4E L",* if $4_h(a, b) = 0$, then *p(a, b) > 0.* ... (4.4)

Next, let *0 e* [0,.5[. Construct *4E L"* such that [for all *i E N*-{h}, q",(y, *x) =* *(z, x) =1 & 4,(Y, z) = 0]* and *[4'b(x* Y) =1 & 4h(x z) = 417(Y, *z) = 0]*. By (4.4), *p"(x,*y)= 1. If p (z, x) > 1-0 , then applying Propositions 3.3.1 and 3.3.2, *ji(z,*y) > 1-0. But by BUR,*/3(z,* y)=1-0, and therefore *p(x, z) > 0* . It then follows by BN and BM that (4.4) extends to:

for all *0* E [0, 1], for all a, be *X* and for all *qEL",* if $q_h(a, b) = 0,$ then *p(a, b) > 0* . ... (4.5)

Since $4_h(a, b) +\ _h(b, a) =1$ and *p(a, b) + p(b, a) = 1,* it is clear that the weak inequality in (4.5) can be replaced by a strict equality. It is also clear the individual *h* is unique, and so

there exists a unique hE Nsuch that for all *0* E [0,1], for all *a, be* and for all *4e V,* if $_h(a, b) = 0$, then *p(a, b) = 0* . ... (4.6)

Now applying Propositions 3.2.1 and 3.1.1, it follows that for all *q* e D_1, P *=4h.* ■

The desire to escape the Arrow paradox has provided one of the motivations for adopting a stochastic framework of social choice. However, Theorem 4.1 shows that if the set of all PCFs satisfying LCA and Reg. constitutes the set of permissible individual PCRs, and if social PCFs are assumed to satisfy LCA and Reg, then the stochastic counterparts of Arrow's independence of irrelevant alternatives and Pareto Criterion again leave us with only trivial, dictatorial PGDFs.

Notes

1 See Definition 2.4 below.
2 For some exceptions see Fishburn and Gehrlein [9] and Intriligator [12] in addition to the papers cited below.
3 For an account of a large section of the relevant literature, the reader may refer to two important surveys - Fishburn [10] and Luce [14].
4 For precise definition of>rationalizability< see Definition 2.4 below. Note that Barbera and Valenciano [4] use certain properties implied by >rationalizability< rather than >rationalizability< itself.
5 McLennan [19] establishes additivity of the power structure given the assumption that there are at least six distinct alternatives.

6 This should not be confused with the entirely different property of independence of irrelevant alternatives due to Arrow [1].

7 Cf. Debreu [7], Chipman [6], and Morgan [15].

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