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### PROJECT

## Determinacy and Large Cardinals

Research over the past two decades has revealed a connection between large cardinal axioms (axioms of Set Theory which go beyond the basic axioms of ZFC) and properties of definable sets of real numbers. The intermediary between large cardinals and the real line is the principle of determinacy, stating the existence of winning strategies in infinite games on natural numbers. On the one hand, this principle has been used as a basis for the study of definable sets of reals. On the other hand, it is known to be closely connected to large cardinal axioms.

While at the Wissenschaftskolleg, I propose to conduct research exploiting the connection between large cardinals and determinacy, to the benefit of both subjects. I plan to work with Ronald B. Jensen (Humboldt University, Berlin) and my co-Fellow Martin Zeman, who are both experts on large cardinals, and with my co-Fellow John Steel, who is an expert on both large cardinals and determinacy.

### Recommended Reading

Neeman, Itay. *The Determinacy of Long Games*. De Gruyter Series in Logic and Its Applications, vol. 7. Berlin: de Gruyter, November 2004.

- "An Introduction to Proofs of Determinacy of Long Games." In *Logic Colloquium '01*. Lecture Notes in Logic, vol. 20. Wellesley, MA: AK Peters, 2005.

- "Games of length  $\omega_1$ ." Submitted.

## Set Theory, Infinite Games, and Strong Axioms

Mathematicians reason about the world by proving theorems, that is deriving results of interest through a careful step by step deduction from simple basic principles. My goals in this talk are first to convey through examples an impression of how this is done, and second to give a very informal introduction to my specific area, set theory.

Set theory originated in the late 19th century in work of Georg Cantor on infinite sets. Cantor gave a precise definition of the notion of size, that applies both to finite and infinite sets, and considered the possibility that not all infinite sets are of the same size. I will start the talk with Cantor's definition, several examples of infinite sets, and Cantor's famous theorem on the size of the continuum, with its proof. (I am not including the statement of the theorem here so as to maintain suspense.) From this I will pass to the notion of cardinality. Finally, for this part of the talk, I will discuss elementary embeddings of the universe of sets, noting that their action takes place only on sets of extremely large size, called large cardinals, far removed from the natural numbers.

In the second part of the talk I will consider infinite games on natural numbers. In this part too I will present a proof. I will prove that certain infinite games are determined, meaning that one of the players in the game has a strategy that is guaranteed to win against all plays by the opponent. The existence of such strategies is a concrete statement on sets of natural numbers.

Seemingly the large cardinals of the first part of the talk have nothing to do with the concrete statements on the existence of winning strategies in games on natural numbers. But it turns out that in fact the two are intricately connected, and this is the starting point for my own work.

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### PUBLICATIONS FROM THE FELLOW LIBRARY

Neeman, Itay (2002)

Optimal proofs of determinacy II

<https://kxp.k10plus.de/DB=9.663/PPNSET?PPN=794267858>

Neeman, Itay (2002)

Inner models in the region of a Woodin limit of Woodin cardinals

<https://kxp.k10plus.de/DB=9.663/PPNSET?PPN=77011394X>